# **Modelling and numerical computation of transient internal damping due to thermal expansion mismatch between matrix and particles in metal matrix composites**

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#### **Abstract**

A new model of the transient internal damping (ID) associated with the emission and movements of dislocations around particles in metal matrix composites (MMCs) is developed. These movements on which the proposed model is based are mainly induced during thermal cycles by the internal stress field around particles, which results from the thermal expansion mismatch between particles and matrix. First, from this thermally induced internal stress field, calculated by the Eshelby method, and the critical shear stress opposing the motion of dislocations in their glide plane in the matrix, the number and positions of punched-out dislocations are determined as a function of temperature. Second, the actual positions due to the superposition on the thermal stress field of the alternating shear stress associated with the pendulum oscillations are calculated by a perturbation method. Then the internal damping is derived from the contribution of the dislocation movements to the inelastic strain over a period of oscillation. The role of the experimental parameters is investigated. This simulated ID is compared with experimental results obtained in the case of aluminium-based MMCs. A good agreement between simulated and experimental IDs is found.

## **1. Introduction**

During the cooling of various metal matrix composites (MMCs) a cooling-rate-dependent broad peak appears in the internal damping (ID) spectrum [1, 2]. In these experiments the ID has been characterized by the logarithmic decrement  $\delta$  of the freely decaying oscillations of a torsion pendulum of the inverted type. The peak is not present when the cooling rate  $\ddot{T}$  is zero and then the ID spectrum is very similar to that observed for unreinforced materials, for which the internal damping is generally cooling rate independent. The difference between the internal dampings measured at  $\ddot{T} \neq 0$  and  $\dot{T}= 0$  is hereafter denoted  $\delta_{\dot{T}}$ . At a given temperature  $\delta_t$  increases with increasing cooling rate or increasing period of pendulum oscillations. In the range  $10^{-6}$ – $10^{-5}$ for the surface shear strain amplitude  $\delta_t$  decreases when the shear strain amplitude is increased. Finally, these phenomena appear stronger as the yield stress of the matrix becomes lower [1, 3].

These phenomena have been explained as follows [1, 2]: On cooling MMCs, dislocation loops are emitted to accommodate the thermal expansion mismatch between particles and matrix. The cooling-rate-dependent ID  $\delta_T$  is attributed to the additional movements of these mobile dislocations induced by the alternating shear stress associated with the pendulum oscillations. However, a simple model based on a linear relationship between the inelastic strain and both the pendulum stress and the thermal strain mismatch is not able to describe all the details of the phenomena [2]. Therefore the aim of this paper is to present a more reliable model for this physical mechanism. After recalling some basic principles about ID, a model of punching out of the thermally emitted dislocation loops during the cooling of an MMC is briefly presented. A perturbation method is used to determine the dislocation loop positions when a mechanical stress is superimposed on the thermal stress. Incorporating these concepts into the ID formulation enables us to compute the ID spectrum. Finally, the effects of the cooling rate and the frequency of oscillations on the thus computed ID are presented.

#### **2. Internal damping formulation**

The logarithmic decrement is related to the energy loss during an oscillation,  $\Delta W$ , by the well-known relation  $\delta \approx \Delta W/2W$ , where W is the maximal elastic energy stored in the sample during 1 cycle of oscillation. Furthermore, the general expression for  $\Delta W$  is

$$
\Delta W = \iiint_{V} \int_{V_1} \int_{\text{cycle}} \text{trace}(\bar{\sigma} \cdot d\bar{\epsilon}_{\text{p}}) \, \mathrm{d}V
$$

where V is the sample volume,  $\bar{\sigma}$  is the applied stress tensor and  $\bar{\epsilon}_p$  is the plastic strain tensor induced by the applied and internal stress fields. For a parallelepiped sample with a thickness very much smaller than its width and for a torque applied along the longitudinal axis of the sample, the general expression for the stress tensor [4] becomes very simple. The only non-zero term varies as the distance between the considered point and the symmetry plane parallel to the largest faces. Then the expressions for  $\Delta W$ , W and  $\delta_T$  are

$$
\Delta W = \frac{2V}{\tau_{\text{max}}}\int_{0}^{\tau_{\text{max}}}\int_{\text{cycle}}\dot{\gamma}_{\text{p}}(\tau)\,\text{d}t\,\text{d}\tau
$$
\n
$$
W = \frac{V}{6}\frac{\tau_{\text{max}}^2}{\mu}\qquad(1)
$$
\n
$$
\delta_T = \frac{3\mu}{\tau_{\text{max}}^3}\int_{0}^{\tau_{\text{max}}}\int_{\text{cycle}}\tau\sin(\omega t)\dot{\gamma}_{\text{p}}(\tau)\,\text{d}t\,\text{d}\tau
$$

where  $\mu$  is the shear modulus,  $\tau_{\text{max}}$  is the shear stress on the largest face of the sample and  $\dot{\gamma}_p$  is the shear strain rate induced by the thermal and mechanical stresses. Then the problem which is involved in any analytical or numerical calculation of the dissipated energy is to determine  $\dot{\gamma}_p$ . Our approach is presented in the subsequent sections.

#### **3. Punching out of dislocation loop modelling**

To determine the positions of dislocation loops on cooling the MMC, we only consider the simplest situation: a spherical particle in an infinite matrix initially free of stress [5]. In this model the thermally induced stress field around a particle, calculated by the Eshelby method [6], is partially relaxed by the emission of interstitial dislocation loops (a fraction of the geometrically necessary dislocations). For the sake of simplicity we represent the interactions of dislocations with obstacles opposing their motion by a local friction stress  $\tau_f$ , assumed to be independent of the dislocation velocity. Then dislocation loop emission occurs when the driving shear stress in the matrix near the particle-matrix interface, which is calculated by also taking into account the partial stress relaxation expected from the potentially emitted dislocation, is greater than  $\tau_f$ . The radius of the circular loops is chosen in order to obtain equality between the Tresca shear stress in the matrix near the interface and the shear stress in the glide direction of the dislocation. After emission the loops glide far away



Fig. 1. Distance  $x^0$  of dislocation groups from particle-matrix interface vs. temperature on cooling a 7075 AI alloy/15 vol.% SiC composite (particle diameter 10  $\mu$ m) from 430 K (stressfree temperature).

from the interface as long as the driving stress remains greater than the friction stress in the matrix. Residual stresses, image effects, dipolar interactions between loops and plastic incompatibilities are taken into account in the calculation of the driving stress. The evolutions of  $\tau_f$  with the temperature and the local dislocation density are included in the model. In addition, note that to fulfil the spherical symmetry of the system, 12 dislocation loops, which are assumed to be equally distributed among the various glide directions of the A1 matrix, are treated simultaneously by grouping and melting them in a somewhat equivalent spherical plastic shell [5]. Results obtained by the Hamann-Fougères model [5] for a 7075 A1 alloy reinforced with 15 vol.% SiC particles are reported in Fig. 1, where the number of dislocation groups and their distances from the particle-matrix interface are plotted *vs.* temperature. From this figure two domains can be distinguished. First, during the beginning of cooling (temperature range 430-320 K) the matrix remains elastic. Second, on further decreasing the temperature, the yield stress of the matrix is exceeded. Then the punching out of dislocation groups occurs with a quasi-constant temperature decrement between successive events.

# **4. Internal damping modelling**

When a periodic shear stress is applied during the temperature variation, the above-calculated dislocation displacements are modified. For the sake of simplicity let us describe the approach that we use to determine this perturbation with just one dislocation loop, a constant friction shear stress and a constant cooling rate. At a temperature  $T$  the relation between the driving shear stress  $F(x^0, T)$  acting on the loop situated at  $x^0$ ("thermal" position without applied shear stress) and the friction shear stress is  $F(x^0, T) = \tau_f$ . When a mechanical periodic shear stress is applied, the equality between the driving and friction shear stresses becomes so by  $F(x^1, T) + \tau \sin(\omega t) = \tau_f$ , where  $x^1$  is the actual position of the dislocation loop that is close to  $x<sup>0</sup>$ . Then a first-order expansion in the position variable and a time derivation enable us to express the difference between the actual and "thermal" dislocation velocities as

$$
\dot{x}^1 - \dot{x}^0 = \tau \frac{|\dot{T}| \sin(\omega t) \frac{\partial^2 F}{\partial x \partial T} - \omega \cos(\omega t) \frac{\partial F}{\partial x}}{\left(\frac{\partial F}{\partial x}\right)^2}
$$
(2)

In fact, this expression is only valid when the driving shear stress is higher than the friction stress or, equivalently, when the dislocation velocity is positive. Under the other circumstance the dislocation velocity is zero. In consideration of this remark, it appears that two regimes must be distinguished for the dislocation movements:

(a) a continuous forward motion of the dislocation if  $\dot{x}^1 > 0$  throughout the period of oscillation (regime "a");

(b) a jerky movement of the dislocation if during the pendulum oscillation the driving shear stress becomes lower than  $\tau_f$  (regime "b").

In the latter case the time lapse during which the dislocation movement occurs is deduced from the condition  $x^1>0$ . Furthermore, the contribution of these dislocation movements to  $\dot{\gamma}_p$  is calculated from  $\dot{x}^1$  in consideration of the following remarks.

(i) The purely thermal displacement (term  $\dot{x}^0$ ) does not contribute to any shear strain, because the corresponding strain is of dilatational type.

(ii) According to the glide direction and the Burgers vector orientation, the velocity of the various segments of a dislocation loop may be increased or moderated by the applied stress, thus leading to some shear strain (in regime *"a"* the right-hand-side term of eqn. (2) is responsible for this shear strain, which tends to follow the applied stress with some phase shift due to the non-linearity of the mechanism, whereas in regime *"b"*  the contribution of the dislocation displacement to the shear strain rate is also derived from  $\dot{x}^1 - \dot{x}^0$ , but in this case its expression is not always given by eqn. (2), since during the immobility time of the dislocation  $\dot{x}^1 - \dot{x}^0 = -\dot{x}^0$ .

(iii) For the sake of simplicity an average orientation factor for the various glide directions, estimated to be  $\frac{1}{3}$ , has been used instead of determining  $\dot{x}^1 - \dot{x}^0$  and its contribution to  $\dot{\gamma}_p$  for each direction of slip.

(iv) Orowan's law can be applied over a cubic volume  $h<sup>3</sup>$  around the particle, where h is the mean distance between particles, to relate  $\dot{x}^1 - \dot{x}^0$  to  $\dot{\gamma}_p$  for each dislocation loop:  $\dot{\gamma}_p = 2\pi \rho(\dot{x}^1 - \dot{x}^0) b/h^3$ , where  $\rho$  is the loop radius.

Finally, to get the  $\delta_t$  value, a time integration over 1 cycle (or the fraction of the cycle during which the velocity is positive) and a subsequent integration over the shear stress range, from the neutral fibre where the shear stress is zero to the specimen surface where it is maximal, have to be carried out. As initially mentioned, the above description of our approach has been given for a single group of emitted dislocations. For real calculations all the groups of dislocation loops (see Fig. 1) must be taken into account. Thus the driving shear stress must be considered as a function of all the dislocation loop positions in order to take the dipolar interactions and plastic incompatibilities



Fig. 2. Internal damping *vs.* absolute temperature for two cooling rates in 7075 Al alloy/15 vol.% SiC composite with a pendulum period of 3.3 s, a maximal shear stress of 0.18 MPa and a cooling rate of (a) 100 and (b) 200 K  $h^{-1}$ : solid curves, computed results for a particle diameter of 10  $\mu$ m; solid squares, experimental data for a particle mean size of 10  $\mu$ m.



Fig. 3. Internal damping maximum *vs.* frequency; a, experimental data; b, computed results (same material and experimental conditions as for Fig. 2 but with a cooling rate of 100 K  $h^{-1}$ ).

into account. The thus computed variations in the internal damping  $\delta_t$  vs. temperature are reported in Fig. 2 for two cooling rates. The experimental values measured under the same experimental conditions are represented by squares. It appears that the agreement between experimental and computed values is good. The shape of the curves is principally governed by two different mechanisms. First, the monotonic increase in  $\delta_r$  observed with decreasing T below the temperature for which the first loop is emitted is due to a regular increase in the dislocation loop number (see Fig. 1). Second, the decrease in  $\delta_T$  for the lowest temperature range results from the decrease in the dislocation mobility in this temperature range. The comparison of the height of the maximum of the ID spectrum *vs.* the oscillation frequency for computed and experimental results also exhibits very good agreement as shown in Fig. 3.

# **5. Conclusions**

The good agreement between experimental data and computed results for the cooling-rate-dependent ID in MMCs confirms the proposed interpretation. This phe**nomenon** results from the additional displacements of the thermally emitted dislocation loops which are induced by the oscillations of the pendulum.

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